

Time Complexity

Lecture 21

Section 9.6

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Wed, Mar 8, 2017

- 1 “Big-O” Notation: $O(f)$
- 2 Estimating Run Times
- 3 Comparison of Growth Rates
- 4 Evaluation of List Implementations
 - Basic Member Functions
 - Search and Sort Functions
- 5 Assignment

Outline

- 1 “Big-O” Notation: $O(f)$
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“Big-O” Notation: $O(f)$

Definition (Big-O)

- Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function.
- A function $g : \mathbb{R} \rightarrow \mathbb{R}$ is $O(f)$ (“big-O of f ”) if there exist constants $c \in \mathbb{R}$, $n_0 \in \mathbb{N}$ such that

$$g(n) \leq cf(n)$$

for all $n \geq n_0$.

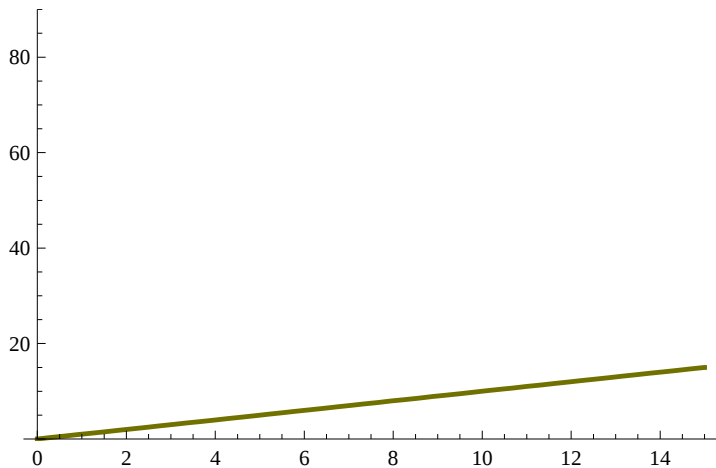
- In other words, $g(n)$ is bounded above by a constant multiple of $f(n)$ from some point on.

Examples

Example (Proving and Disproving Big-O)

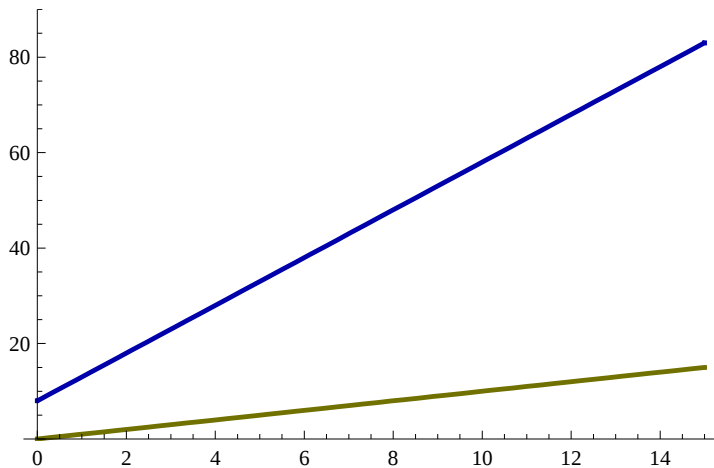
- Show that $5n + 8$ is $O(n)$.
- Show that $n^2 + 10n + 5$ is $O(n^2)$.
- Show that $n^2 + 10n + 5$ is *not* $O(n)$.

Examples



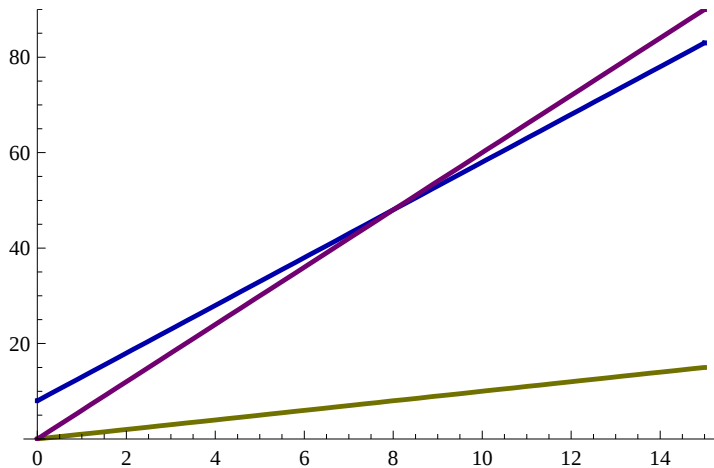
$$f(n) = n$$

Examples



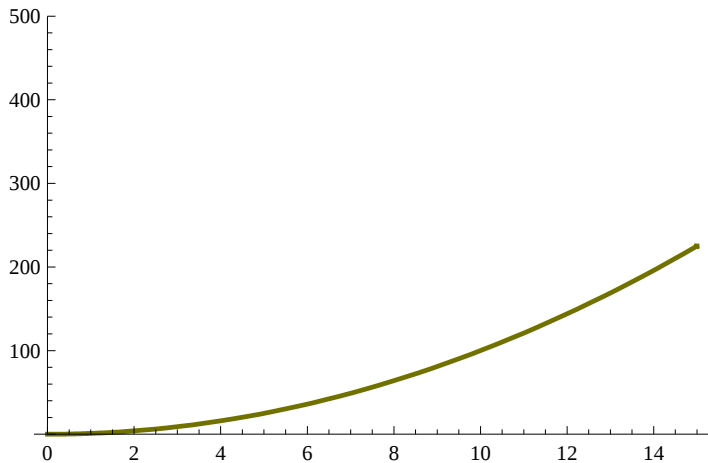
$$f(n) = n \text{ and } g(n) = 5n + 8$$

Examples



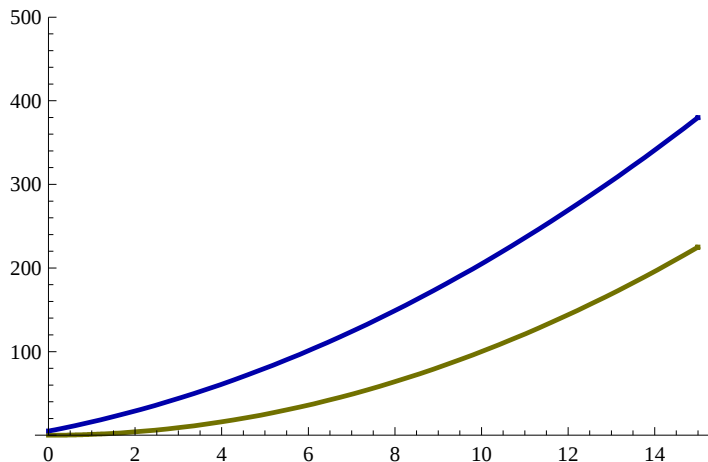
$$f(n) = n \text{ and } g(n) = 5n + 8 \text{ and } 6f(n) = 6n$$

Examples



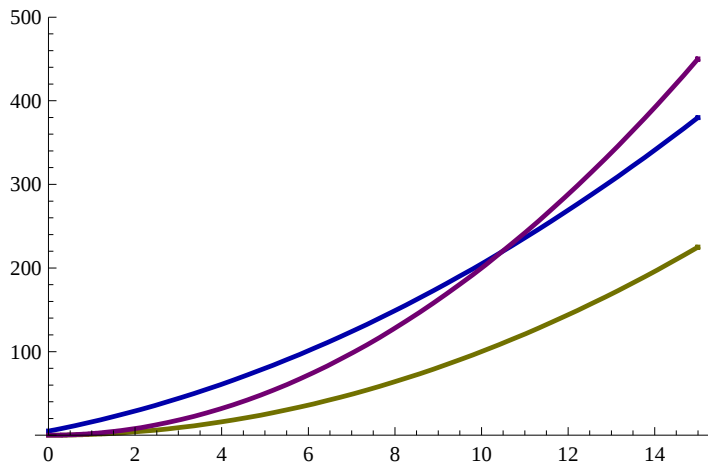
$$f(n) = n^2$$

Examples



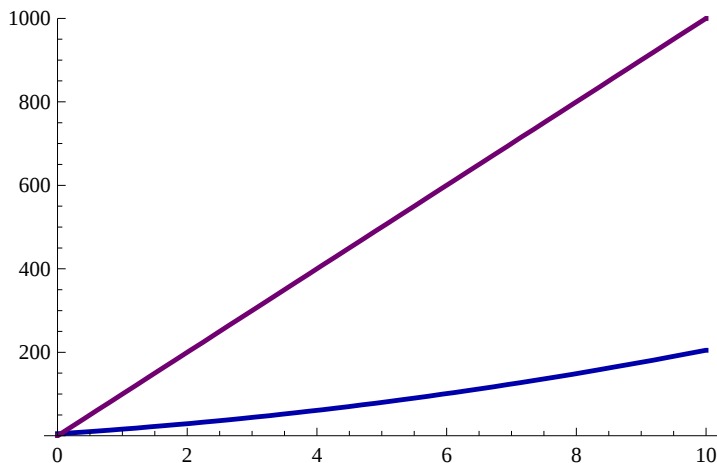
$$f(n) = n^2 \text{ and } g(n) = n^2 + 10n + 5$$

Examples



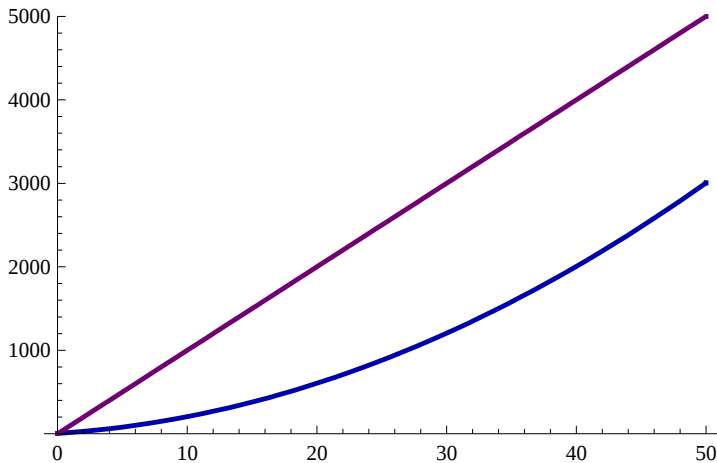
$$f(n) = n^2 \text{ and } g(n) = n^2 + 10n + 5 \text{ and } 2f(n) = 2n^2$$

Examples



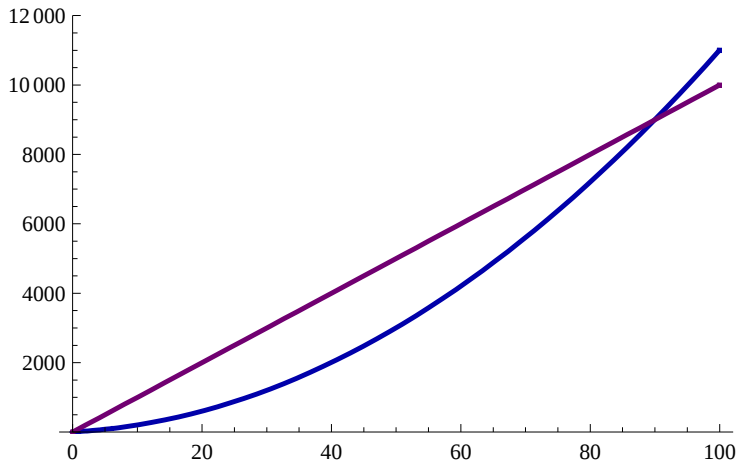
$100f(n) = 100n$ and $g(n) = n^2 + 10n + 5$, over $0 \leq n \leq 10$

Examples



$100f(n) = 100n$ and $g(n) = n^2 + 10n + 5$, over $0 \leq n \leq 50$

Examples



$100f(n) = 100n$ and $g(n) = n^2 + 10n + 5$, over $0 \leq n \leq 100$

Growth Rates

- If g is $O(f)$, then the growth rate of $g(n)$ is no greater than the growth rate of $f(n)$.
- If g is $O(f)$ and f is $O(g)$, then $f(n)$ and $g(n)$ have the same growth rate.
- In this case, we say that g is $\Theta(f)$ (big-theta).

Example

Example (Equal Growth Rates)

- Show that $5n + 8$ is $\Theta(n)$.

Equal Growth Rates

Theorem (Equal Growth Rates)

- $an + b$ is $\Theta(n)$ for all real numbers a and b with $a > 0$.
- $an^2 + bn + c$ is $\Theta(n^2)$ for all real numbers a , b , and c with $a > 0$.
- $an^3 + bn^2 + cn + d$ is $\Theta(n^3)$ for all real numbers a , b , c , and d with $a > 0$.
- And so on.

Common Growth Rates

Growth Rate	Example
$\Theta(1)$	Access an array element
$\Theta(\log_2 n)$	Binary search
$\Theta(n)$	Sequential search
$\Theta(n \log_2 n)$	Merge sort
$\Theta(n^2)$	Selection sort
$\Theta(2^n)$	Factor an integer
$\Theta(n!)$	Traveling salesman problem

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Estimating Run Times

- Suppose the run time of a program is $\Theta(f)$, for a specific function $f(n)$.
- Then the run time $t(n)$ satisfies

$$c_1 f(n) \leq t(n) \leq c_2 f(n)$$

for some real numbers c_1 and c_2 .

- We typically assume that $t(n) = cf(n)$ for some real number c .

Estimating Run Times

- Suppose program runs in t_0 seconds when the input size is n_0 .
Then

$$c = \frac{t_0}{f(n_0)}.$$

- Thus,

$$t(n) = t_0 \left(\frac{f(n)}{f(n_0)} \right).$$

Example

Example (Estimating Run Times)

- Suppose the run time of a program is $\Theta(n^2)$.

Example

Example (Estimating Run Times)

- Suppose the run time of a program is $\Theta(n^2)$.
- Suppose further that the program runs in $t_0 = 5 \mu\text{sec}$ when the input size is $n_0 = 100$.

Example

Example (Estimating Run Times)

- Suppose the run time of a program is $\Theta(n^2)$.
- Suppose further that the program runs in $t_0 = 5 \mu\text{sec}$ when the input size is $n_0 = 100$.
- Then

$$t(n) = 5 \left(\frac{n^2}{100^2} \right) \mu\text{sec}.$$

Example

Example (Estimating Run Times)

- Suppose the run time of a program is $\Theta(n^2)$.
- Suppose further that the program runs in $t_0 = 5 \mu\text{sec}$ when the input size is $n_0 = 100$.
- Then

$$t(n) = 5 \left(\frac{n^2}{100^2} \right) \mu\text{sec}.$$

- Thus, if the input size is 1000, then the run time is

$$t(1000) = 5 \left(\frac{1000^2}{100^2} \right) = 500 \mu\text{sec}.$$

Example

Example (Estimating Run Times)

- Suppose the run time of a program is

$$\Theta(n \log n).$$

Example

Example (Estimating Run Times)

- Suppose the run time of a program is

$$\Theta(n \log n).$$

- Suppose further that the program runs in $t_0 = 5 \mu\text{sec}$ when the input size is $n_0 = 100$.

Example

Example (Estimating Run Times)

- Suppose the run time of a program is

$$\Theta(n \log n).$$

- Suppose further that the program runs in $t_0 = 5 \mu\text{sec}$ when the input size is $n_0 = 100$.
- Then

$$t(n) = 5 \left(\frac{n \log n}{100 \log 100} \right) \mu\text{sec}.$$

Example

Example (Estimating Run Times)

- Suppose the run time of a program is

$$\Theta(n \log n).$$

- Suppose further that the program runs in $t_0 = 5 \mu\text{sec}$ when the input size is $n_0 = 100$.
- Then

$$t(n) = 5 \left(\frac{n \log n}{100 \log 100} \right) \mu\text{sec}.$$

- Thus, if the input size is 1000, then the run time is

$$t(n) = 5 \left(\frac{1000 \log 1000}{100 \log 100} \right) = 75 \mu\text{sec}.$$

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Comparison of Growth Rates

- Consider programs with run times that are $\Theta(1)$, $\Theta(\log n)$, $\Theta(n)$, $\Theta(n \log n)$, and $\Theta(n^2)$.
- Assume that each program runs in 1 μsec when the input size is 100.
- Calculate the run times for input sizes 10^2 , 10^3 , 10^4 , 10^5 , 10^6 , 10^7 , and 10^8 .

Comparison of Growth Rates

n	$\Theta(1)$	$\Theta(\log_2 n)$	$\Theta(n)$	$\Theta(n \log_2 n)$	$\Theta(n^2)$
10^2	1 μ sec	1 μ sec	1 μ sec	1 μ sec	1 μ sec

Comparison of Growth Rates

n	$\Theta(1)$	$\Theta(\log_2 n)$	$\Theta(n)$	$\Theta(n \log_2 n)$	$\Theta(n^2)$
10^2	1 μsec	1 μsec	1 μsec	1 μsec	1 μsec
10^3	1 μsec	1.5 μsec	10 μsec	15 μsec	100 μsec

Comparison of Growth Rates

n	$\Theta(1)$	$\Theta(\log_2 n)$	$\Theta(n)$	$\Theta(n \log_2 n)$	$\Theta(n^2)$
10^2	1 μ sec	1 μ sec	1 μ sec	1 μ sec	1 μ sec
10^3	1 μ sec	1.5 μ sec	10 μ sec	15 μ sec	100 μ sec
10^4	1 μ sec	2 μ sec	100 μ sec	200 μ sec	10 msec

Comparison of Growth Rates

n	$\Theta(1)$	$\Theta(\log_2 n)$	$\Theta(n)$	$\Theta(n \log_2 n)$	$\Theta(n^2)$
10^2	1 μ sec	1 μ sec	1 μ sec	1 μ sec	1 μ sec
10^3	1 μ sec	1.5 μ sec	10 μ sec	15 μ sec	100 μ sec
10^4	1 μ sec	2 μ sec	100 μ sec	200 μ sec	10 msec
10^5	1 μ sec	2.5 μ sec	1 msec	2.5 msec	1 sec

Comparison of Growth Rates

n	$\Theta(1)$	$\Theta(\log_2 n)$	$\Theta(n)$	$\Theta(n \log_2 n)$	$\Theta(n^2)$
10^2	1 μ sec	1 μ sec	1 μ sec	1 μ sec	1 μ sec
10^3	1 μ sec	1.5 μ sec	10 μ sec	15 μ sec	100 μ sec
10^4	1 μ sec	2 μ sec	100 μ sec	200 μ sec	10 msec
10^5	1 μ sec	2.5 μ sec	1 msec	2.5 msec	1 sec
10^6	1 μ sec	3 μ sec	10 msec	30 msec	1.7 min

Comparison of Growth Rates

n	$\Theta(1)$	$\Theta(\log_2 n)$	$\Theta(n)$	$\Theta(n \log_2 n)$	$\Theta(n^2)$
10^2	1 μ sec	1 μ sec	1 μ sec	1 μ sec	1 μ sec
10^3	1 μ sec	1.5 μ sec	10 μ sec	15 μ sec	100 μ sec
10^4	1 μ sec	2 μ sec	100 μ sec	200 μ sec	10 msec
10^5	1 μ sec	2.5 μ sec	1 msec	2.5 msec	1 sec
10^6	1 μ sec	3 μ sec	10 msec	30 msec	1.7 min
10^7	1 μ sec	3.5 μ sec	100 msec	350 msec	2.8 hr

Comparison of Growth Rates

n	$\Theta(1)$	$\Theta(\log_2 n)$	$\Theta(n)$	$\Theta(n \log_2 n)$	$\Theta(n^2)$
10^2	1 μ sec	1 μ sec	1 μ sec	1 μ sec	1 μ sec
10^3	1 μ sec	1.5 μ sec	10 μ sec	15 μ sec	100 μ sec
10^4	1 μ sec	2 μ sec	100 μ sec	200 μ sec	10 msec
10^5	1 μ sec	2.5 μ sec	1 msec	2.5 msec	1 sec
10^6	1 μ sec	3 μ sec	10 msec	30 msec	1.7 min
10^7	1 μ sec	3.5 μ sec	100 msec	350 msec	2.8 hr
10^8	1 μ sec	4 μ sec	1 sec	4 sec	11.7 d

Comparison of $\Theta(n^2)$ and $\Theta(2^n)$

- Now consider a program with growth rate $\Theta(2^n)$.
- Assume that the program runs in 1 μ sec when the input size is 100.
- Calculate the run times for input sizes 100, 110, 120, 130, 140, 150, and 160.
- Note that we will not try to go as far as 10^3 . How come?

Comparison of $\Theta(n^2)$ and $\Theta(2^n)$

n	$\Theta(n^2)$	$\Theta(2^n)$
100	1 μ sec	1 μ sec

Comparison of $\Theta(n^2)$ and $\Theta(2^n)$

n	$\Theta(n^2)$	$\Theta(2^n)$
100	1 μ sec	1 μ sec
110	1.2 μ sec	1 msec

Comparison of $\Theta(n^2)$ and $\Theta(2^n)$

n	$\Theta(n^2)$	$\Theta(2^n)$
100	1 μ sec	1 μ sec
110	1.2 μ sec	1 msec
120	1.4 μ sec	1 sec

Comparison of $\Theta(n^2)$ and $\Theta(2^n)$

n	$\Theta(n^2)$	$\Theta(2^n)$
100	1 μ sec	1 μ sec
110	1.2 μ sec	1 msec
120	1.4 μ sec	1 sec
130	1.7 μ sec	18 min

Comparison of $\Theta(n^2)$ and $\Theta(2^n)$

n	$\Theta(n^2)$	$\Theta(2^n)$
100	1 μ sec	1 μ sec
110	1.2 μ sec	1 msec
120	1.4 μ sec	1 sec
130	1.7 μ sec	18 min
140	2.0 μ sec	13 d

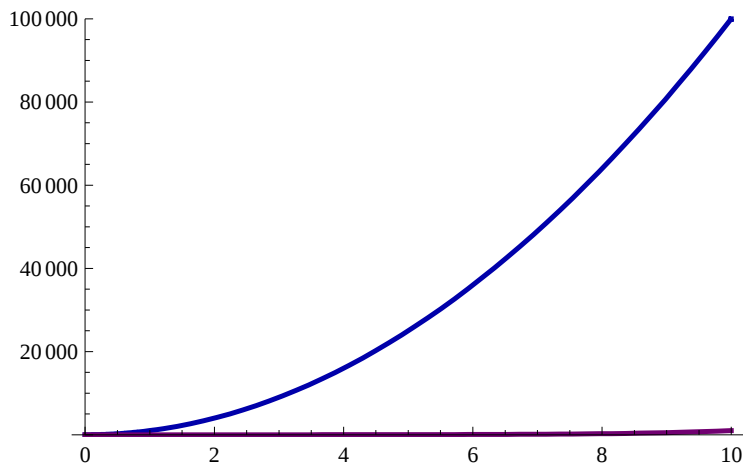
Comparison of $\Theta(n^2)$ and $\Theta(2^n)$

n	$\Theta(n^2)$	$\Theta(2^n)$
100	1 μ sec	1 μ sec
110	1.2 μ sec	1 msec
120	1.4 μ sec	1 sec
130	1.7 μ sec	18 min
140	2.0 μ sec	13 d
150	2.3 μ sec	37 yr

Comparison of $\Theta(n^2)$ and $\Theta(2^n)$

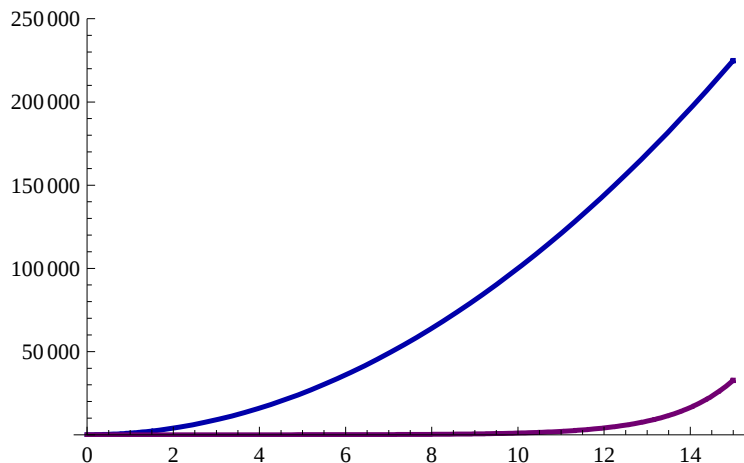
n	$\Theta(n^2)$	$\Theta(2^n)$
100	1 μ sec	1 μ sec
110	1.2 μ sec	1 msec
120	1.4 μ sec	1 sec
130	1.7 μ sec	18 min
140	2.0 μ sec	13 d
150	2.3 μ sec	37 yr
160	2.6 μ sec	37,000 yr

Examples



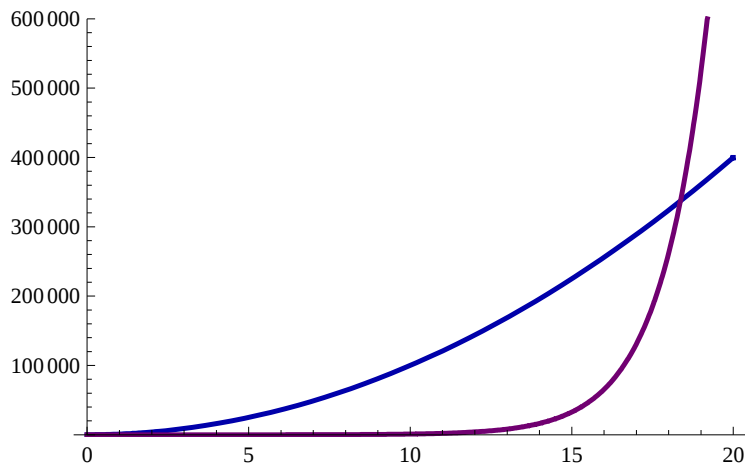
$$g(n) = 2^n \text{ vs. } 1000f(n) = 1000n^2, 0 \leq n \leq 10$$

Examples



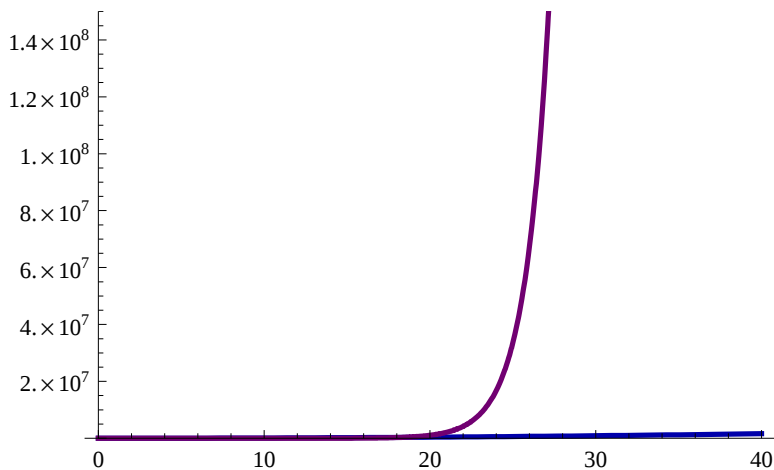
$$g(n) = 2^n \text{ vs. } 1000f(n) = 1000n^2, 0 \leq n \leq 15$$

Examples



$$g(n) = 2^n \text{ vs. } 1000f(n) = 1000n^2, 0 \leq n \leq 20$$

Examples



$$g(n) = 2^n \text{ vs. } 1000f(n) = 1000n^2, 0 \leq n \leq 40$$

Comparison of $\Theta(2^n)$ and $\Theta(n!)$

- Now consider a program with growth rate $\Theta(n!)$.
- Assume that the program runs in 1 μ sec when the input size is 100.
- Calculate the run times for input sizes 100, 101, 102, $\dots$, 110.
- Note that we will not try to go as far as 120. How come?

Comparison of $\Theta(2^n)$ and $\Theta(n!)$

n	$\Theta(2^n)$	$\Theta(n!)$
100	1 μ sec	1 μ sec

Comparison of $\Theta(2^n)$ and $\Theta(n!)$

n	$\Theta(2^n)$	$\Theta(n!)$
100	1 μ sec	1 μ sec
101	2 μ sec	100 μ sec

Comparison of $\Theta(2^n)$ and $\Theta(n!)$

n	$\Theta(2^n)$	$\Theta(n!)$
100	1 μ sec	1 μ sec
101	2 μ sec	100 μ sec
102	4 μ sec	10 msec

Comparison of $\Theta(2^n)$ and $\Theta(n!)$

n	$\Theta(2^n)$	$\Theta(n!)$
100	1 μ sec	1 μ sec
101	2 μ sec	100 μ sec
102	4 μ sec	10 msec
103	8 μ sec	1.1 sec

Comparison of $\Theta(2^n)$ and $\Theta(n!)$

n	$\Theta(2^n)$	$\Theta(n!)$
100	1 μ sec	1 μ sec
101	2 μ sec	100 μ sec
102	4 μ sec	10 msec
103	8 μ sec	1.1 sec
104	16 μ sec	1.8 min

Comparison of $\Theta(2^n)$ and $\Theta(n!)$

n	$\Theta(2^n)$	$\Theta(n!)$
100	1 μ sec	1 μ sec
101	2 μ sec	100 μ sec
102	4 μ sec	10 msec
103	8 μ sec	1.1 sec
104	16 μ sec	1.8 min
105	32 μ sec	3.2 hr

Comparison of $\Theta(2^n)$ and $\Theta(n!)$

n	$\Theta(2^n)$	$\Theta(n!)$
100	1 μ sec	1 μ sec
101	2 μ sec	100 μ sec
102	4 μ sec	10 msec
103	8 μ sec	1.1 sec
104	16 μ sec	1.8 min
105	32 μ sec	3.2 hr
106	64 μ sec	14 d

Comparison of $\Theta(2^n)$ and $\Theta(n!)$

n	$\Theta(2^n)$	$\Theta(n!)$
100	1 μ sec	1 μ sec
101	2 μ sec	100 μ sec
102	4 μ sec	10 msec
103	8 μ sec	1.1 sec
104	16 μ sec	1.8 min
105	32 μ sec	3.2 hr
106	64 μ sec	14 d
107	128 μ sec	4.2 yr

Comparison of $\Theta(2^n)$ and $\Theta(n!)$

n	$\Theta(2^n)$	$\Theta(n!)$
100	1 μ sec	1 μ sec
101	2 μ sec	100 μ sec
102	4 μ sec	10 msec
103	8 μ sec	1.1 sec
104	16 μ sec	1.8 min
105	32 μ sec	3.2 hr
106	64 μ sec	14 d
107	128 μ sec	4.2 yr
108	256 μ sec	450 yr

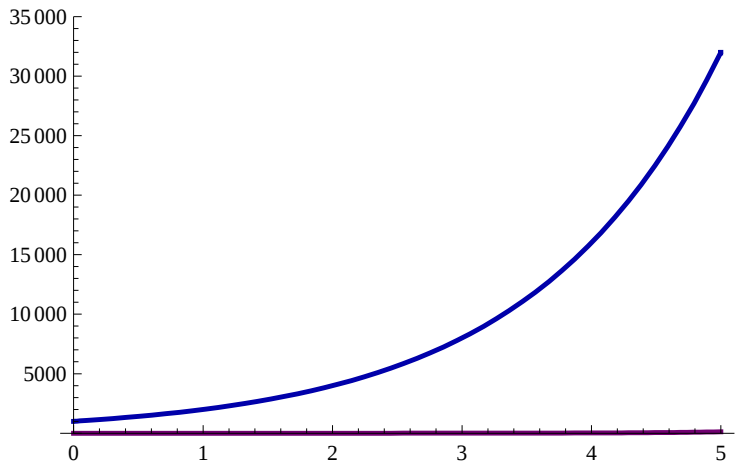
Comparison of $\Theta(2^n)$ and $\Theta(n!)$

n	$\Theta(2^n)$	$\Theta(n!)$
100	1 μ sec	1 μ sec
101	2 μ sec	100 μ sec
102	4 μ sec	10 msec
103	8 μ sec	1.1 sec
104	16 μ sec	1.8 min
105	32 μ sec	3.2 hr
106	64 μ sec	14 d
107	128 μ sec	4.2 yr
108	256 μ sec	450 yr
109	512 μ sec	49,000 yr

Comparison of $\Theta(2^n)$ and $\Theta(n!)$

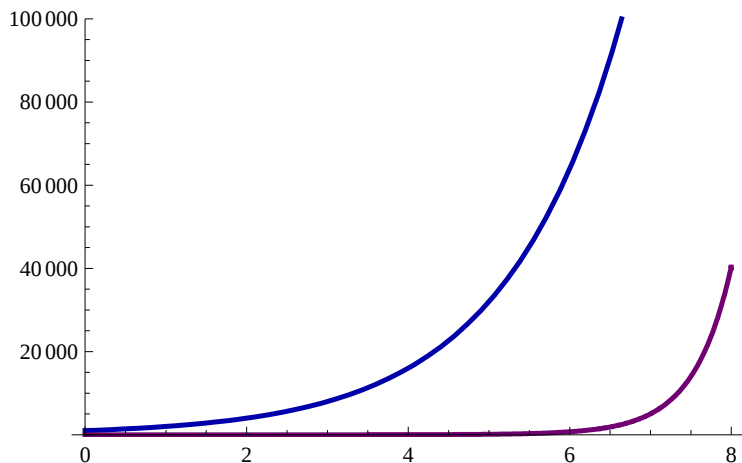
n	$\Theta(2^n)$	$\Theta(n!)$
100	1 μ sec	1 μ sec
101	2 μ sec	100 μ sec
102	4 μ sec	10 msec
103	8 μ sec	1.1 sec
104	16 μ sec	1.8 min
105	32 μ sec	3.2 hr
106	64 μ sec	14 d
107	128 μ sec	4.2 yr
108	256 μ sec	450 yr
109	512 μ sec	49,000 yr
110	1024 μ sec	5,400 mill

Examples



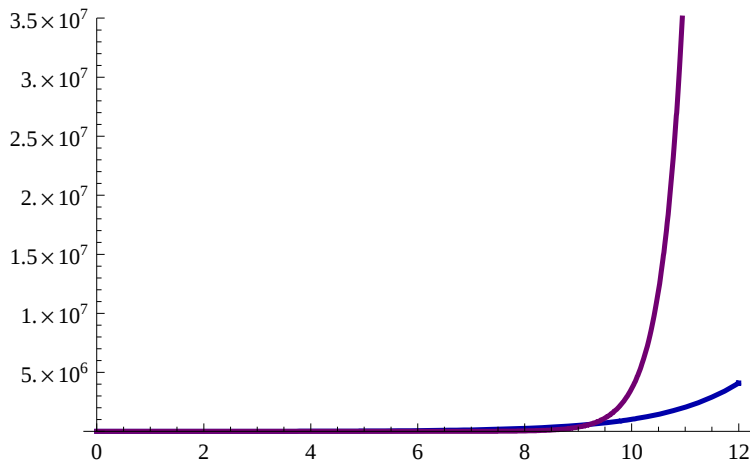
$$g(n) = n! \text{ vs. } 1000f(n) = 1000 \cdot 2^n, 0 \leq n \leq 5$$

Examples



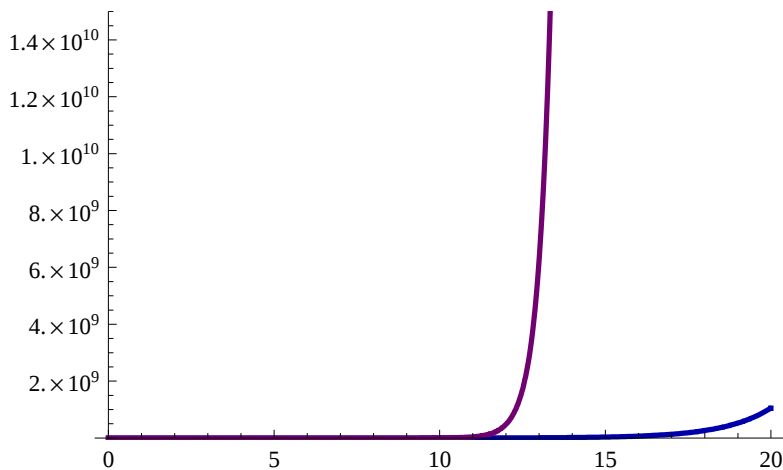
$$g(n) = n! \text{ vs. } 1000f(n) = 1000 \cdot 2^n, 0 \leq n \leq 8$$

Examples



$$g(n) = n! \text{ vs. } 1000f(n) = 1000 \cdot 2^n, 0 \leq n \leq 12$$

Examples



$$g(n) = n! \text{ vs. } 1000f(n) = 1000 \cdot 2^n, 0 \leq n \leq 20$$

Example

Example (Traveling Salesman)

- `TravelingSalesman.cpp`.

Feasible vs. Infeasible

Definition (Feasible)

An algorithm is considered to be **feasible** if it is $O(n^k)$ for some integer k . Otherwise, it is considered to be **infeasible**.

- Of course, nearly every algorithm is “feasible” for small input sizes.
- It is feasible to factor small integers (< 50 digits).
- It is infeasible to factor large integers (> 200 digits).

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Basic Member Functions

- I tested the functions `getElement()`, `insert()`, `remove()`, `pushFront()`, `popFront()`, `pushBack()`, **and** `popBack()`.

Evaluation of Lists

	operator[]	insert	remove
ArrayList	0.05 μ sec	1.01 μ sec	1.02 μ sec
CircArrayList	0.09 μ sec	1.14 μ sec	1.14 μ sec
LinkedList	0.33 μ sec	1.76 μ sec	1.82 μ sec
LinkedListwTail	0.29 μ sec	1.72 μ sec	1.87 μ sec
DoublyLinkedList	0.21 μ sec	1.37 μ sec	1.60 μ sec
CircLinkedList	0.20 μ sec	1.46 μ sec	1.63 μ sec

	pushFront	popFront	pushBack	popBack
ArrayList	1.56 μ sec	1.49 μ sec	0.31 μ sec	0.38 μ sec
CircArrayList	0.33 μ sec	0.30 μ sec	0.32 μ sec	0.31 μ sec
LinkedList	1.00 μ sec	1.39 μ sec	2.04 μ sec	3.36 μ sec
LinkedListwTail	0.97 μ sec	1.26 μ sec	0.85 μ sec	3.23 μ sec
DoublyLinkedList	1.05 μ sec	1.44 μ sec	1.06 μ sec	1.45 μ sec
CircLinkedList	1.12 μ sec	1.45 μ sec	1.11 μ sec	1.45 μ sec

List size = 100.

Evaluation of Lists

	operator[]	insert	remove
ArrayList	0.04 μ sec	5.94 μ sec	6.13 μ sec
CircArrayList	0.08 μ sec	3.68 μ sec	3.73 μ sec
LinkedList	3.29 μ sec	10.7 μ sec	10.5 μ sec
LinkedListwTail	3.21 μ sec	10.7 μ sec	11.1 μ sec
DoublyLinkedList	1.19 μ sec	5.41 μ sec	5.73 μ sec
CircLinkedList	1.14 μ sec	5.54 μ sec	5.86 μ sec

	pushFront	popFront	pushBack	popBack
ArrayList	11.7 μ sec	11.5 μ sec	0.20 μ sec	0.28 μ sec
CircArrayList	0.23 μ sec	0.21 μ sec	0.20 μ sec	0.21 μ sec
LinkedList	1.06 μ sec	1.43 μ sec	17.0 μ sec	31.3 μ sec
LinkedListwTail	1.13 μ sec	1.41 μ sec	0.96 μ sec	31.3 μ sec
DoublyLinkedList	1.17 μ sec	1.54 μ sec	1.15 μ sec	1.53 μ sec
CircLinkedList	1.24 μ sec	1.54 μ sec	1.20 μ sec	1.52 μ sec

List size = 1000.

Evaluation of Lists

	operator[]	insert	remove
ArrayList	0.04 μ sec	67.4 μ sec	61.5 μ sec
CircArrayList	0.08 μ sec	31.1 μ sec	29.9 μ sec
LinkedList	36.9 μ sec	111. μ sec	116. μ sec
LinkedListwTail	36.5 μ sec	112. μ sec	116. μ sec
DoublyLinkedList	18.6 μ sec	58.3 μ sec	61.0 μ sec
CircLinkedList	18.2 μ sec	59.8 μ sec	60.6 μ sec

	pushFront	popFront	pushBack	popBack
ArrayList	127. μ sec	114. μ sec	0.22 μ sec	0.29 μ sec
CircArrayList	0.24 μ sec	0.21 μ sec	0.22 μ sec	0.22 μ sec
LinkedList	1.14 μ sec	1.44 μ sec	165. μ sec	314. μ sec
LinkedListwTail	1.03 μ sec	1.45 μ sec	0.98 μ sec	314. μ sec
DoublyLinkedList	1.27 μ sec	1.59 μ sec	1.26 μ sec	1.64 μ sec
CircLinkedList	1.34 μ sec	1.55 μ sec	1.30 μ sec	1.68 μ sec

List size = 10000.

Evaluation of Lists

	operator[]	insert	remove
ArrayList	0.04 μ sec	1.01 msec	977. μ sec
CircArrayList	0.08 μ sec	172. μ sec	179. μ sec
LinkedList	381. μ sec	499. μ sec	524. μ sec
LinkedListwTail	381. μ sec	495. μ sec	516. μ sec
DoublyLinkedList	188. μ sec	422. μ sec	427. μ sec
CircLinkedList	183. μ sec	409. μ sec	420. μ sec

	pushFront	popFront	pushBack	popBack
ArrayList	1.17 msec	1.14 msec	0.21 μ sec	0.28 μ sec
CircArrayList	0.23 μ sec	0.21 μ sec	0.21 μ sec	0.22 μ sec
LinkedList	1.09 μ sec	1.47 μ sec	2.86 msec	5.59 msec
LinkedListwTail	1.06 μ sec	1.39 μ sec	0.98 μ sec	5.61 msec
DoublyLinkedList	1.18 μ sec	1.61 μ sec	1.19 μ sec	1.61 μ sec
CircLinkedList	1.21 μ sec	1.62 μ sec	1.20 μ sec	1.63 μ sec

List size = 100000.

Outline

- 1 “Big-O” Notation: $O(f)$
- 2 Estimating Run Times
- 3 Comparison of Growth Rates
- 4 Evaluation of List Implementations**
 - Basic Member Functions
 - Search and Sort Functions
- 5 Assignment

Evaluation of Lists

	Seq Search	Bin Search	seqSearch	binSearch
ArrayList	616 μ sec	94.4 μ sec	31.3 μ sec	13.5 μ sec
CircArrayList	810 μ sec	127 μ sec	36.5 μ sec	16.7 μ sec
LinkedList	1.66 msec	351 μ sec	39.5 μ sec	63.6 μ sec
DoublyLinkedList	1.45 msec	258 μ sec	39.3 μ sec	64.0 μ sec
CircLinkedList	1.44 msec	249 μ sec	40.3 μ sec	62.9 μ sec

	Sel Sort	Merge Sort	selSort	mergeSort
ArrayList	675 μ sec	644 μ sec	31.1 μ sec	268 μ sec
CircArrayList	895 μ sec	799 μ sec	37.6 μ sec	322 μ sec
LinkedList	2.70 msec	1.77 msec	35.9 μ sec	641 μ sec
DoublyLinkedList	1.59 msec	1.44 msec	36.5 μ sec	646 μ sec
CircLinkedList	1.59 msec	1.53 msec	36.7 μ sec	756 μ sec

List size = 100.

Evaluation of Lists

	Seq Search	Bin Search	seqSearch	binSearch
ArrayList	61.5 msec	1.34 msec	2.56 msec	174 μ sec
CircArrayList	79.6 msec	1.85 msec	2.61 msec	173 μ sec
LinkedList	1.01 sec	45.5 msec	3.83 msec	5.87 msec
DoublyLinkedList	660 msec	20.5 msec	3.81 msec	5.93 msec
CircLinkedList	626 msec	20.0 msec	4.03 msec	5.82 msec

	Sel Sort	Merge Sort	selSort	mergeSort
ArrayList	62.1 msec	7.88 msec	2.37 msec	3.37 msec
CircArrayList	81.0 msec	10.0 msec	3.10 msec	4.15 msec
LinkedList	2.38 sec	154 msec	3.74 msec	9.44 msec
DoublyLinkedList	645 msec	62.5 msec	3.60 msec	9.67 msec
CircLinkedList	616 msec	62.4 msec	3.83 msec	10.6 msec

List size = 1000.

Evaluation of Lists

	Seq Search	Bin Search	seqSearch	binSearch
ArrayList	6.01 sec	17.4 msec	252. msec	2.13 msec
CircArrayList	7.96 sec	25.6 msec	253. msec	2.10 msec
LinkedList	20.7 min	7.21 sec	417. msec	735. msec
DoublyLinkedList	15.7 min	3.88 sec	405. msec	726. msec
CircLinkedList	15.1 min	3.81 sec	434. msec	721. msec

	Sel Sort	Merge Sort	selSort	mergeSort
ArrayList	6.02 sec	102. msec	230. msec	41.6 msec
CircArrayList	7.97 sec	130. msec	320. msec	52.1 msec
LinkedList	41.3 min	20.5 sec	402. msec	134. msec
DoublyLinkedList	15.5 min	9.93 sec	392. msec	136. msec
CircLinkedList	15.7 min	9.92 sec	423. msec	147. msec

List size = 10000.

Outline

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Assignment

Assignment

- Read Section 9.6.